Some Probabilistic viewpoints & tools for Collective Dynamics and Partial Differential Equations

L. Della Maestra

DeVinci Research Center, Pôle Léonard de Vinci, 92916 Paris La Défense, France

contact: laetitia.della_maestra ⊕ devinci.fr

The aim of this talk is to review the links between certain mean-field interacting particle systems, stochastic differential equations and partial differential equations from mathematical physics [1, 2, 3], and their implications.

On the one hand, we will see how optimal transport tools and coupling methods can help with studying these partial differential equations. On the other hand, we will show how this probabilistic framework naturally provides Monte-Carlo type numerical simulation methods of the solutions of these partial differential equations, as well as statistical tools for solving inverse problems related to them.

Finally, we will see that these techniques are a promising way to achieve a mathematical understanding of deep neural networks [4].

References

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