A stochastic variational principle for a two-fluid model arising in fusion plasma physics

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In this work, we propose a stochastic variational principle [1, 2] for a model of two electrically charged, viscous fluids, describing ions and electrons in a plasma, subject to electromagnetic fields, and under the quasi-neutrality constraint. Having in mind applications to the scrape-off layer (SOL) of fusion plasmas [3], we consider the magnetic field fixed, $B(t, x) = B_0(x)$, and the electric field purely potential, $E(t, x) = -\nabla_x \phi(t, x)$. The equations of motion for the ion and electron fluids coupled by the quasi-neutrality constraint have the mathematical structure of a saddle-point problem, which is amenable to a mixed variational formulation [5]. Eventually, this approach might provide a valid alternative to drift-reduced Braginskii models commonly used to describe particle and energy transport in the SOL plasma [4]. However, in complex geometries mesh-free methods are preferred [6], and this motivates our study of a variational formulation, which could help the derivation of modern particle methods.

First, we show that, without dissipative effects such as particle diffusion, viscosity, and heat fluxes, the model admits a Lagrangian and a corresponding Euler-Poincaré reduced variational principle. In the Lagrangian formulation, the main variables are the flows $\Phi_t(x)$ and $\Psi_t(x)$ describing the displacement of the ion and electron fluid. For any point x, the curve $t \mapsto \Phi_t(x)$ is a Lagrangian trajectory of the ion fluid, while at any time t, Φ_t is an element of the group $\mathrm{Diff}(\Omega)$ of diffeomorphisms [7] of the spatial domain Ω; analogously for Ψ_t. The quasi-neutrality constraint imposes a relation between Φ_t and Ψ_t, which is shown to define a closed submanifold of $\mathrm{Diff}(\Omega) \times \mathrm{Diff}(\Omega)$. In the Euler-Poincaré reduced formulation the main variables are the velocities of the ion and electron fluids. The quasi-neutrality constraint is imposed by means of a Lagrange multiplier, which physically amounts to the potential ϕ. Dissipative effects are then accounted for by adding white noise to the Lagrangian trajectories, in the Euler-Poincaré formulation, following closely the work by Chen, Cruzeiro and Ratiu [2].

References

[1] A. B. Cruzeiro, *Water* **12**, 864 (2020).

[2] X. Chen, A. B. Cruzeiro and T. Ratiu, *J. Nonlinear Sci.* **33**, 5 (2023).

[3] P. Stangeby, *The Plasma Boundary of Magnetic Fusion Devices*, Taylor and Francis, 2000.

[4] R. Schneider, et al, *Contrib. Plasma Phys.* **46**, 3 (2006).

[5] J. V. Gutiérrez-Santacreu, O. Maj and M. Restelli, *J. Comput. Phys.* **373**, 811 (2018).

[6] H. Frerichs, et al, *Comput. Phys. Commun.* **181**, 61 (2010).

[7] D. Ebin and J. Marsden, *Annals of Mathematics* **92**, 102 (1970).